$$PE_g = mgh$$

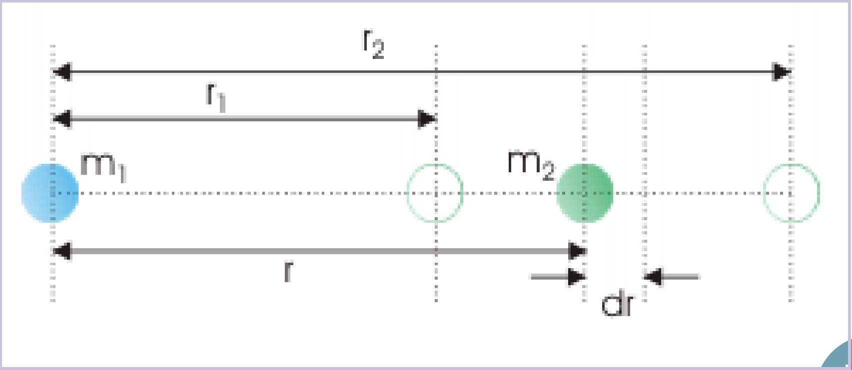
Gravitational Potential Energy

Gravitational Potential Energy

- The force of gravitational attraction between the two masses, at any separation distance r, is given by:
- To increase the separation of the two masses from r_1 to r_2 requires work to be done to overcome their force of attraction. As a result of this work being done, the gravitational potential energy of the system increases.

$$F_G = \frac{Gm_1 m_2}{r^2}$$

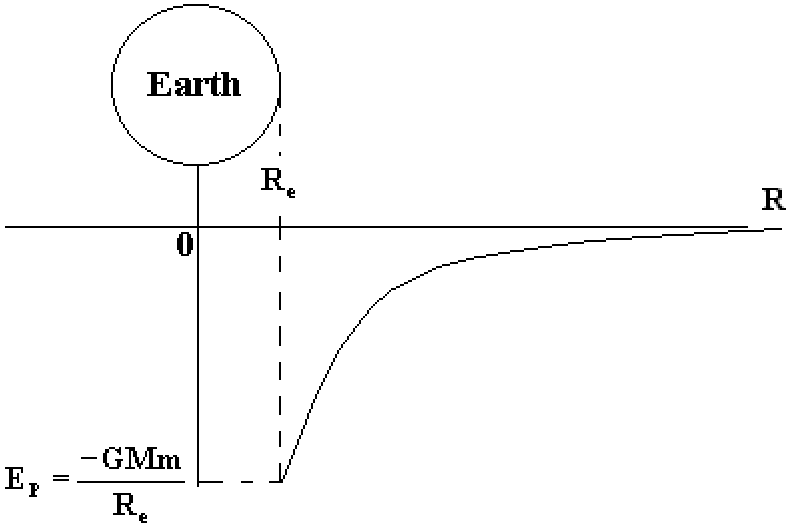
$$G = 6.7x10^{-11} \frac{Nm^2}{kg^2}$$



Gravitational potential energy

- The value of the increase of potential energy (or work done) can be calculated using a force separation graph or an expression derived from the force separation graph
- As r increases, the Eg increases by becoming less negative. In fact as r approaches infinity the Eg reaches zero.

$$U = -\frac{GMm}{r}$$



Examples

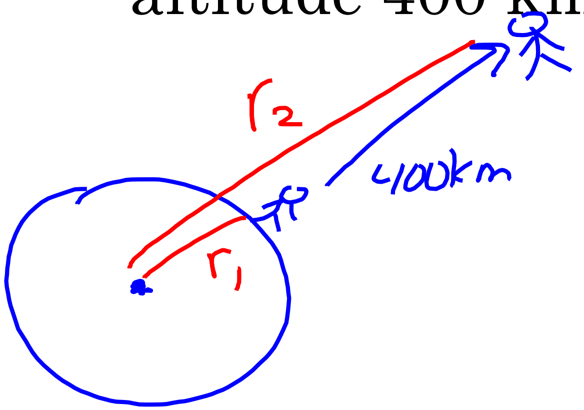
- http://www.youtube.com/watch?v=4Nziux6sejU
- What is the change in gravitational potential energy of a 60 kg astronaut, lifted from the surface of the Earth into a circular orbit of altitude 400 km?

$$\Delta E_g = E_{g2} - E_{g1}$$

$$= -\frac{Gm_1m_2}{r_2} - -\frac{Gm_1m_2}{r_1}$$

$$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(60)}{(6.78\times10^{6})} + \frac{(6.67\times10^{-11})(5.98\times10^{24})(60)}{(6.38\times10^{6})}$$

$$= 2.2\times10^{8}\ \text{J}$$

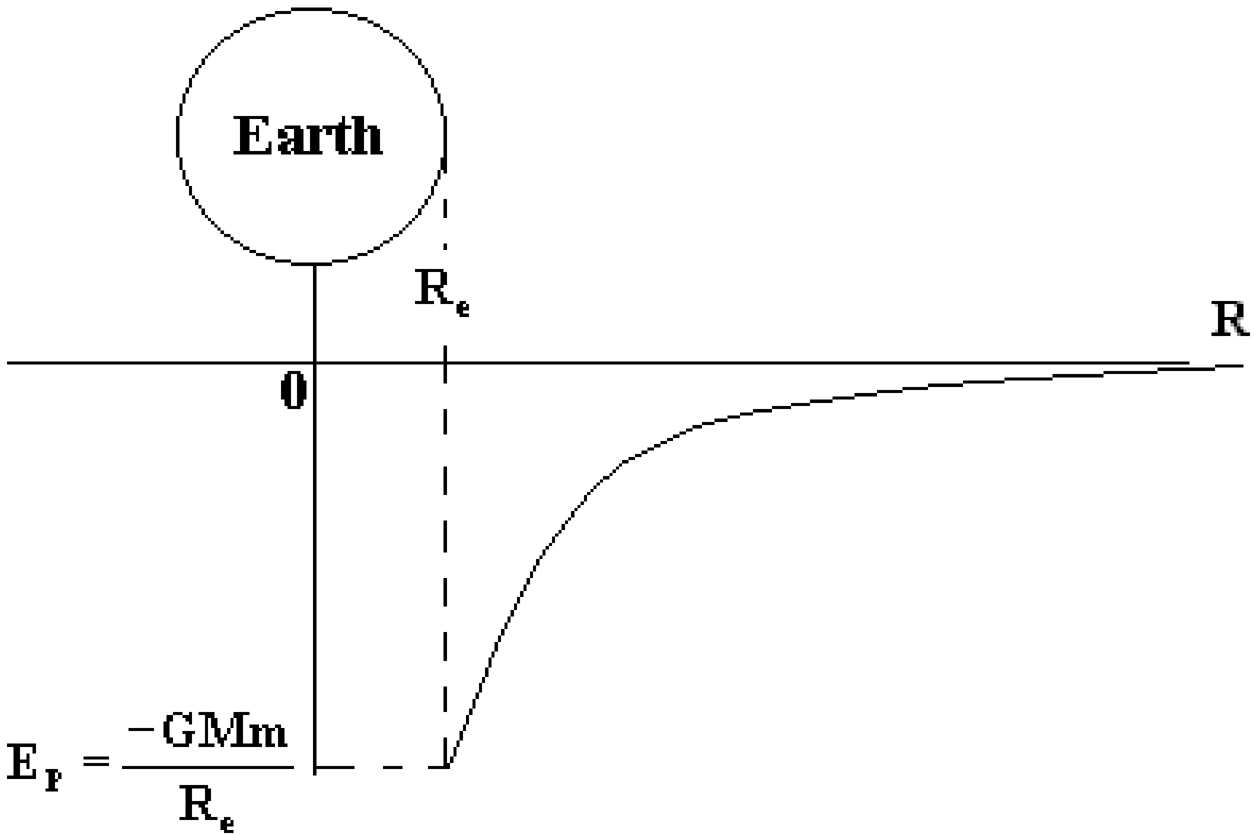


$r_2 = 6.38\times10^6 + 400{,}000$

Escape velocity

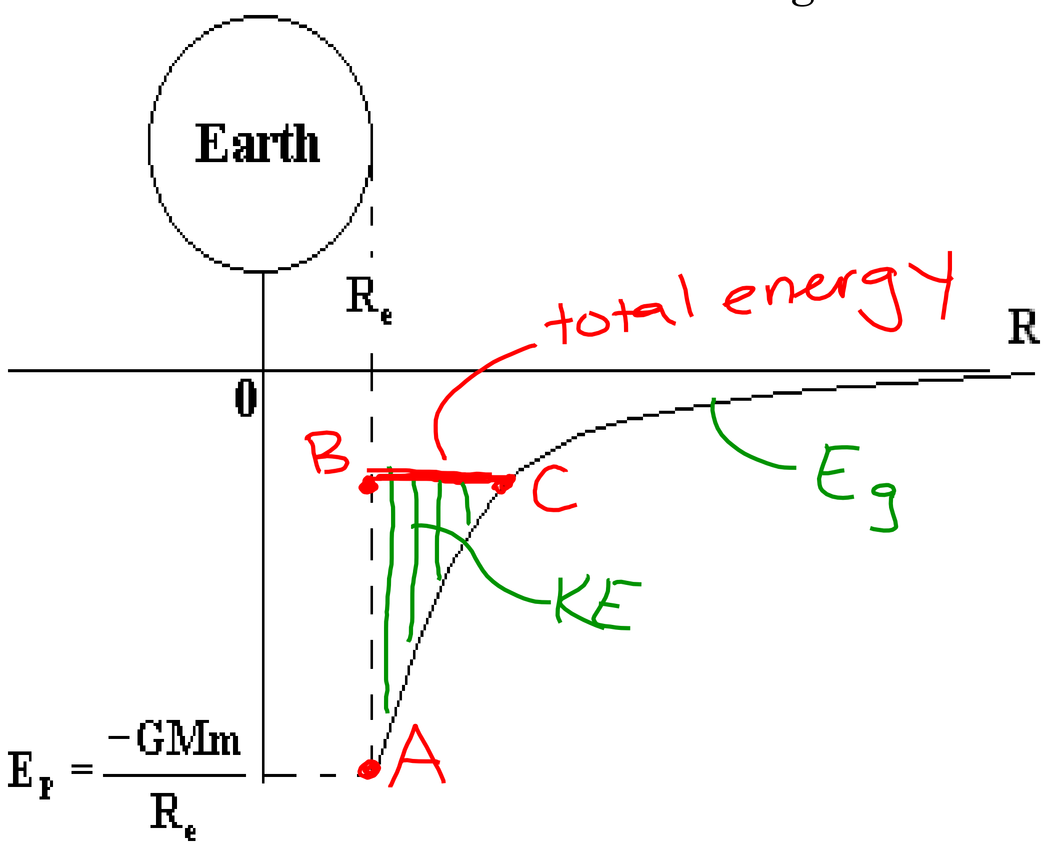
ESCAPE VELOCITY

- The negative value is characteristic of a "potential well" concept



Potential energy of a rocket into space

- Point A: Rocket is at rest
- Point B: Rocket is given a velocity KE
- Point C: Rocket's maximum height



- As the rocket rises, U increases along curve AC
- KE decreases and at any point is given by the length of the vertical line from the curve to the horizontal line BC
- E_{TOT} remains constant along the line BC

Escape velocity

$KE > E_g$

- The initial KE of the rocket must be greater than the potential well at the Earth's surface. The rocket must reach an infinite distance, where U=0J, before nearly coming to rest. Therefore, the E_{TOT} will be positive, since KE>U
- $1/2mv^2 > GMm/r$

$$\frac{1}{2}\cancel{m}v^2 > \frac{GM\cancel{m}}{r}$$

$$\sqrt{v^2} > \sqrt{\frac{2GM}{r}}$$

$$\boxed{v > \sqrt{\frac{2GM}{r}}}$$

Energy value of a earth-satellite system

- A satellite moving in a circular orbit at a constant velocity has a total energy of:

$$\boxed{E_{TOT} = -\frac{1}{2} E_g}$$

- Therefore, the KE required to release a satellite (binding energy), is:

$$\boxed{E = \frac{1}{2} \frac{GMm}{r}}$$

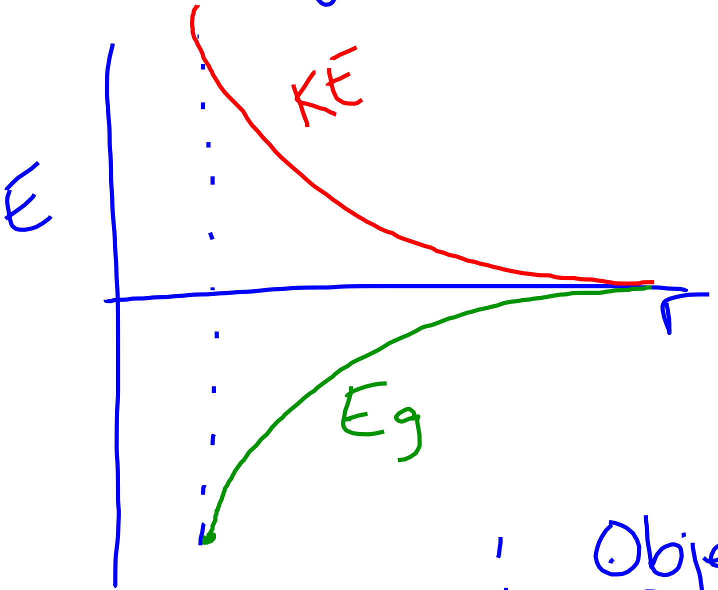
SUMMARY

- The total energy of any object in the Earth's gravitational field is composed of kinetic energy and gravitational potential energy.

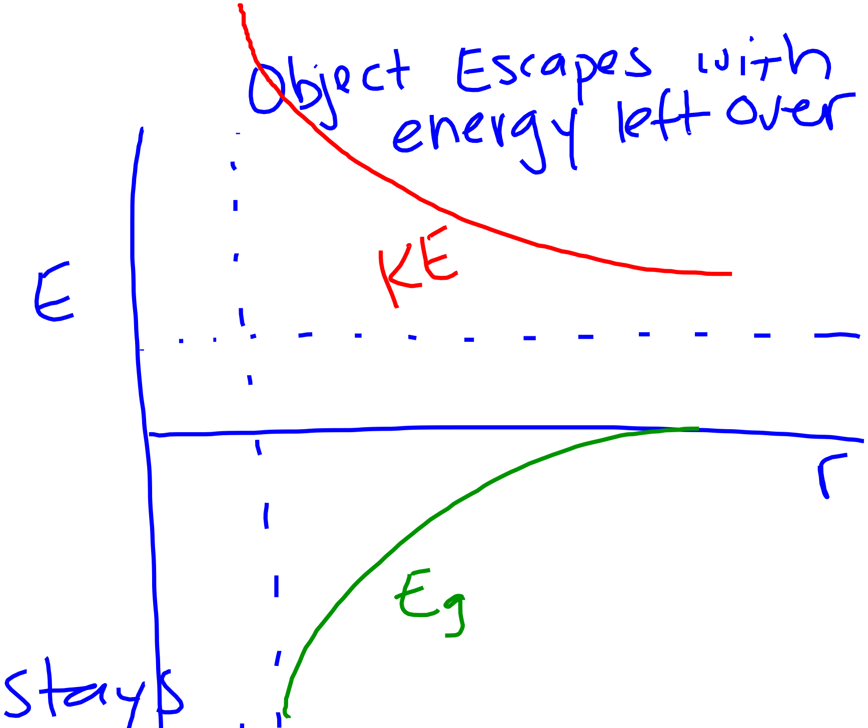
$$E_{TOT} = E_g + KE$$

$$-\frac{GMm}{r} + \frac{1}{2}mv^2$$

Object Just Escapes



Object Escapes with energy left over



Objects Stays Bound to Earth